Vector Clocks, Causally Ordered Multicast

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Lecture 10

Slide Credits: Maarten van Steen

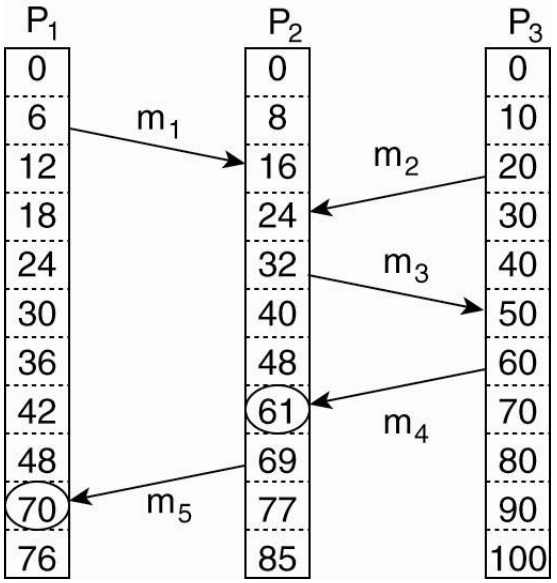
Topics for Today

- (Mattern) Vector Clocks
- Causally Ordered Multicast

Source: TvS 6.2-6.4

Vector Clocks

Observation: Lamport's clocks do not guarantee that if $C(a) < C(b)$ that a **causally preceded** b:



Observation:

Event a: m_1 is received at $T = 16$.

Event b: m_2 is sent at $T = 20$.

We **cannot** conclude that a causally precedes b.

Vector Clocks

Solution:

- Each process P_i has an array $VC_i[1..n]$, where $VC_i[j]$ denotes the number of events that process P_i knows have taken place at process P_j
- When P_i sends a message m, it adds 1 to $VC_i[i]$, and sends VC_i along with m as **vector timestamp** $vt(m)$. Result: upon arrival, recipient knows P_i's timestamp.
- When a process P_j delivers a message m that it received from P_i with vector timestamp $ts(m)$, it
  1) updates each $VC_j[k]$ to $\max\{VC_j[k], ts(m)[k]\}$ for each k
  2) increments $VC_j[j]$ by 1.

Question: What does $VC_i[j] = k$ mean in terms of messages sent and received?

Vector and Lamport Clocks

Lamport Clocks

Rule 1: Each process has its own version of the global clock

Rule 2: Each process increments its global clock version when it performs an internal event or sends a message (which includes a timestamp)

Rule 3: When a process receives a message from another process it updates its global clock version **if the received timestamp is larger.**

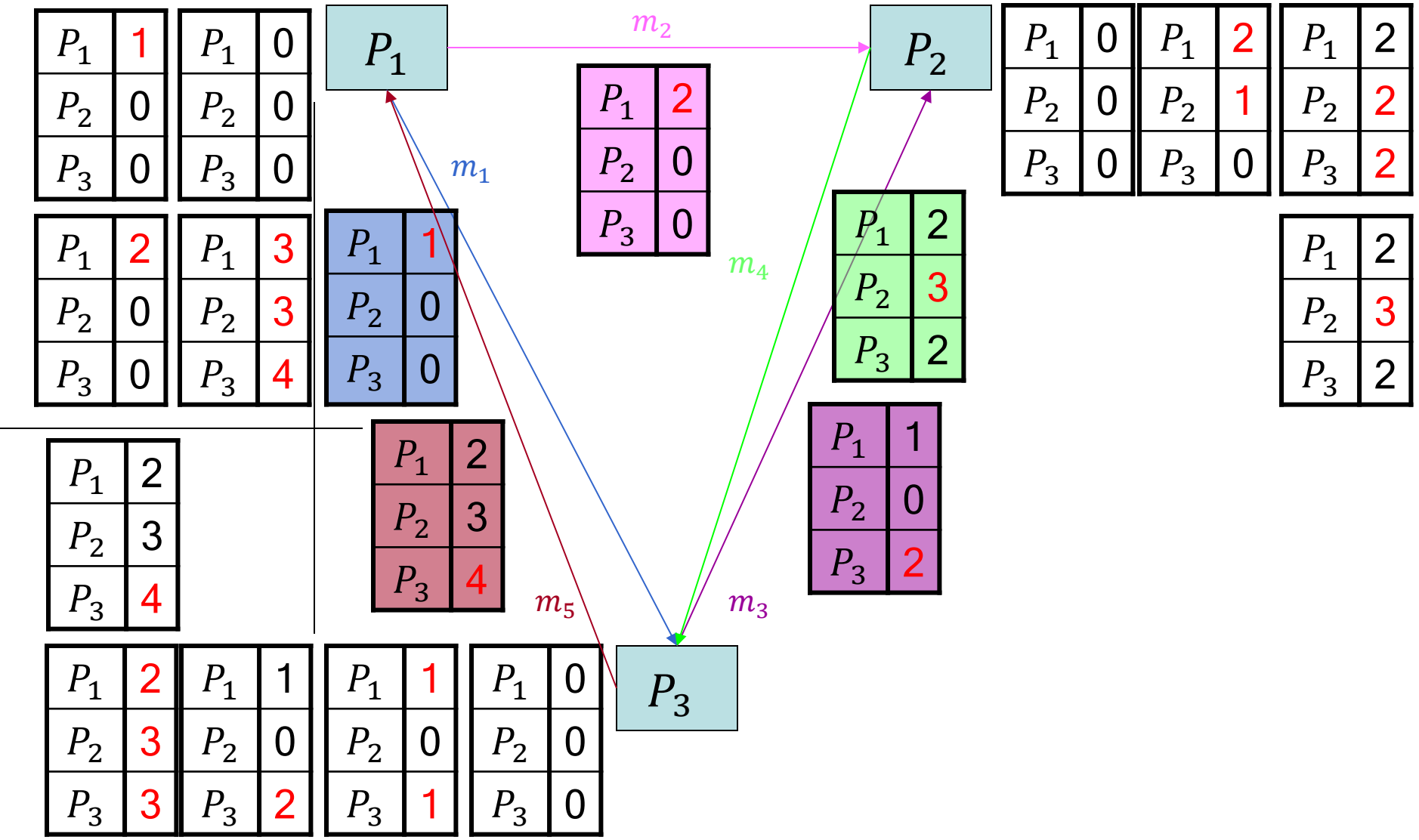
Vector clocks

Rule 1: Each process has its own clock and a version of every other processes' clock.

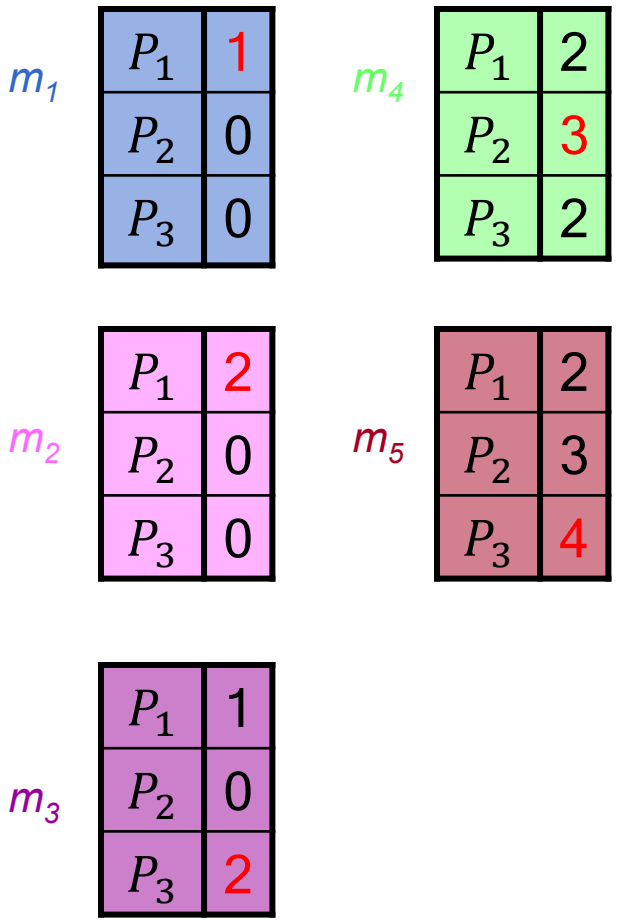
Rule 2: Each process increments its own clock when it sends or receives a message.

Rule 3: When a process receives a message from another process it updates its version of the other clocks' timestamps **if the received timestamp is larger**

Vector Clock Example



Vector Clock Example



1. $m_1 < m_2$
2. $m_1 < m_3$
3. $m_1 < m_4$
4. $m_1 < m_5$

5. $m_2 <> m_3$
6. $m_2 < m_4$
7. $m_2 < m_5$

8. $m_3 < m_4$
9. $m_3 < m_5$

10. $m_4 < m_5$

So Far

- (Mattern) Vector Clocks
- Causally Ordered Multicast

Causally Ordered Multicasting

Observation: We can now ensure that a message is delivered only if all causally preceding messages have already been delivered.

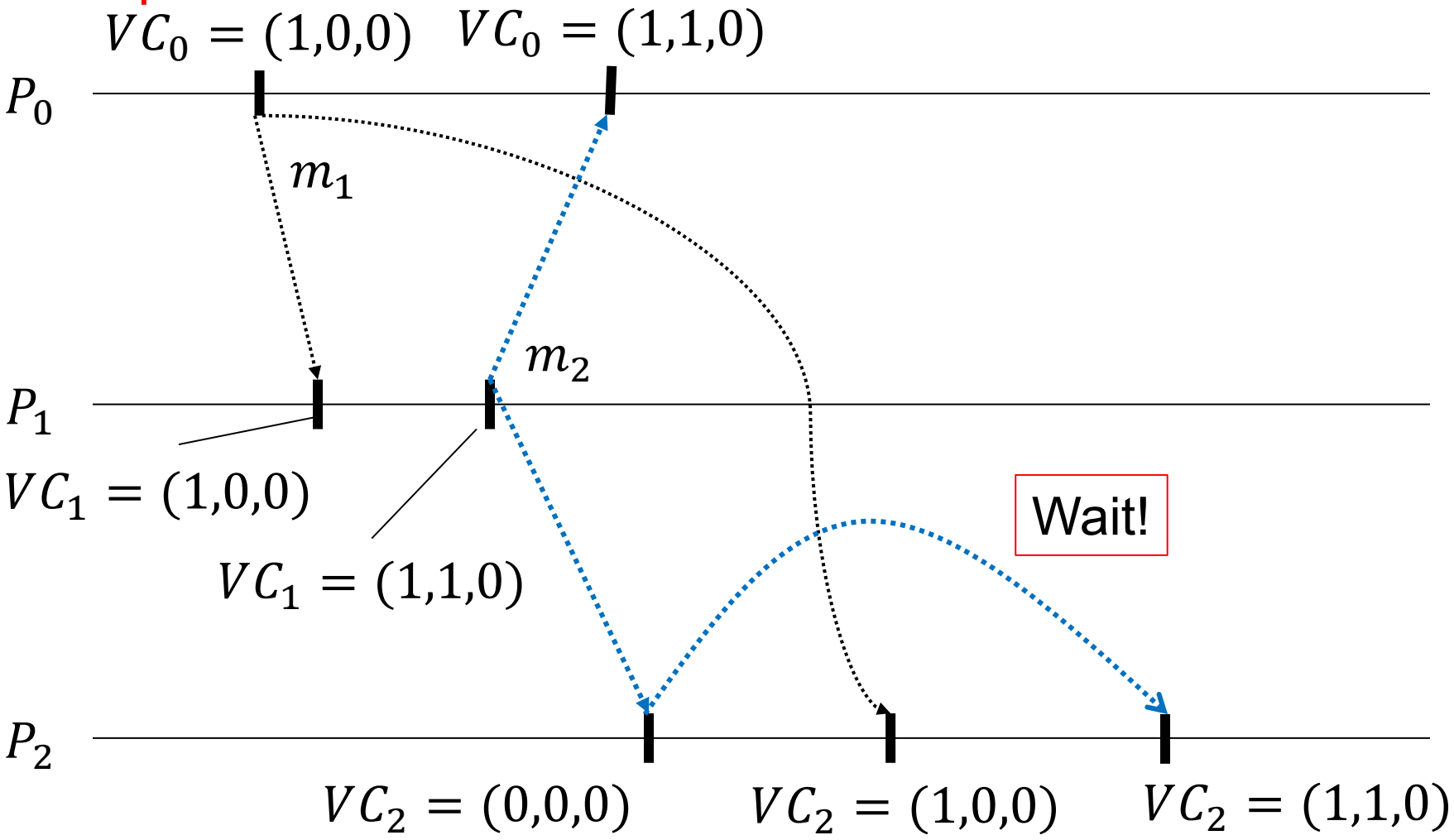
Adjustment: P_i increments $VC_i[i]$ only when sending a message, and P_j "adjusts" VC_j when receiving a message (i.e., effectively does not change $VC_j[j]$).

P_j postpones delivery of m until:

- $ts(m)[i] = VC_j[i] + 1$.
- $ts(m)[k] \leq VC_j[k]$ for $k \neq i$

Causally Ordered Multicasting

Example 1:



Causally Ordered Multicasting

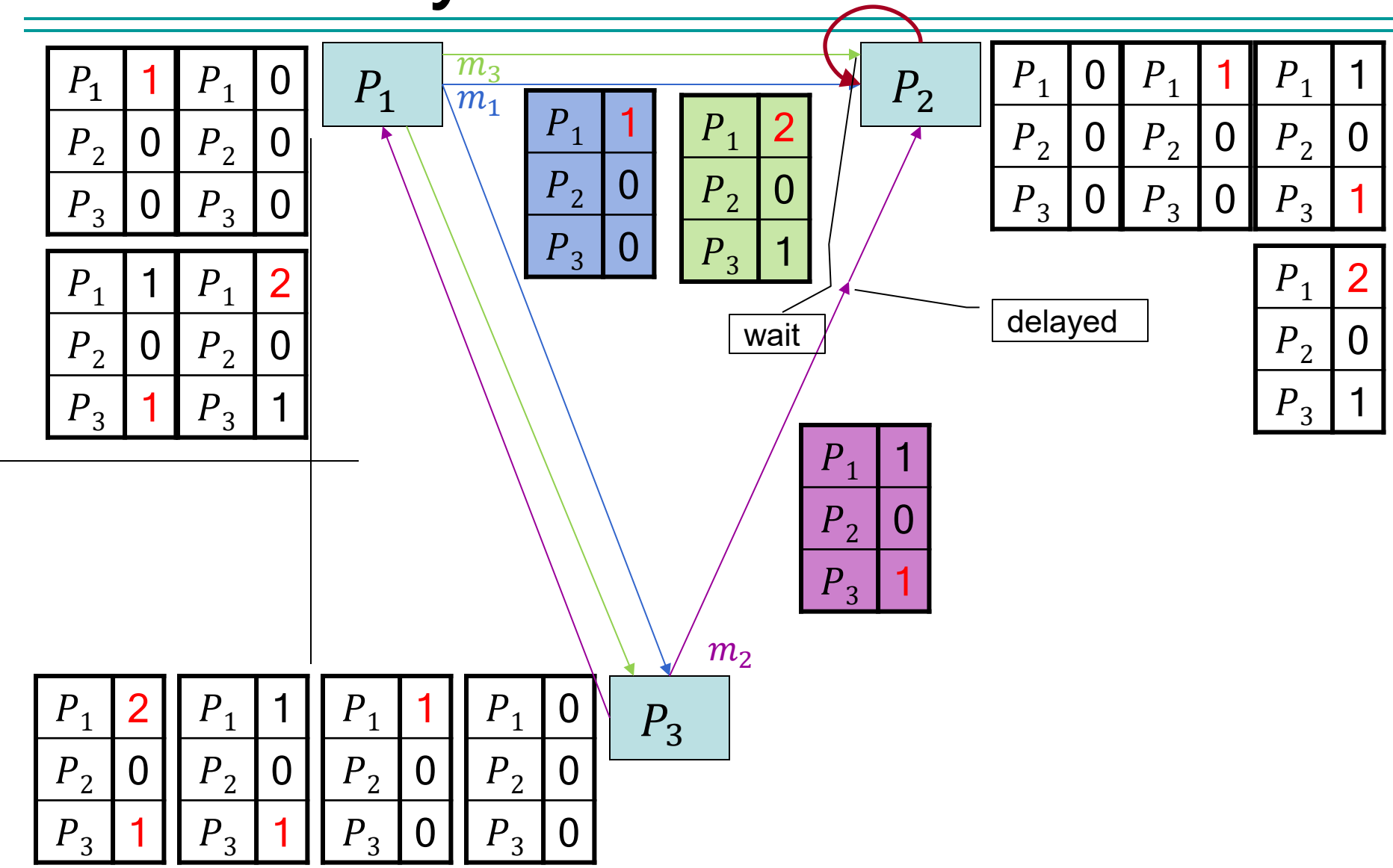
Example 2:

Take $\mathrm{VC}_2 = (0,2,2)$ at P_2

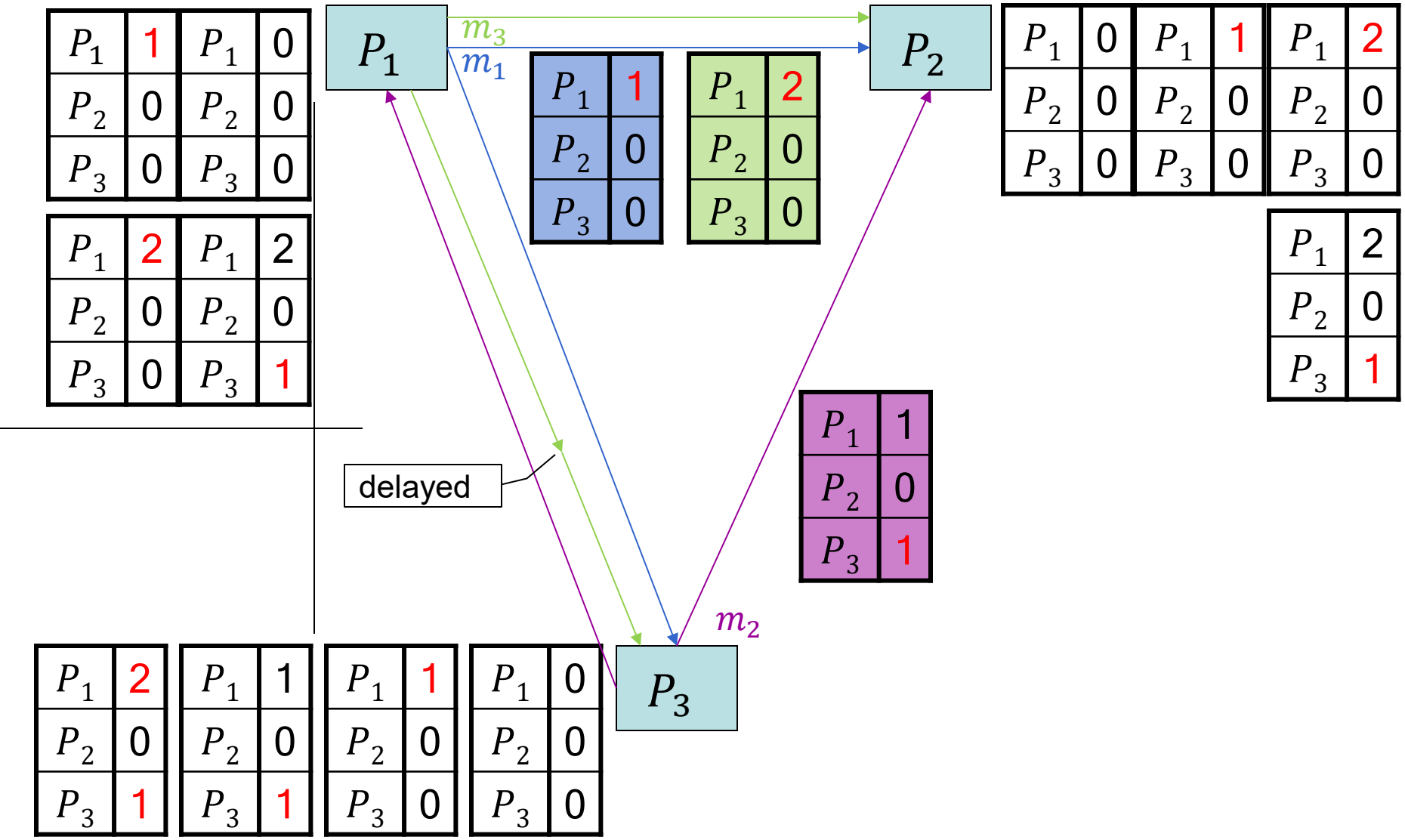
P_2 receives a message with $ts(m) = (1,3,0)$ from P_0.

What information does P_2 have, and what will it do when it receives m (from P_0)?

Causally Ordered Multicast 1



Causally Ordered Multicast 2



Vector Clocks and COM

Vector clocks

Rule 1: Each process has its own clock and a version of every other processes' clock.

Rule 2: Each process increments its own clock when it sends or receives a message.

Rule 3: When a process receives a message from another process it updates its version of the other clocks' timestamps **if the received timestamp is larger**

Causally Ordered Multicast

Rule 1: Each process has its own clock and a version of every other processes' clock.

Rule 2: Each process increments its own clock when it sends a message.

Rule 3: When a process receives a message from another process it updates its version of the sender's timestamp.

Rule 4: A message is delivered only if it is "next in line":

1. It's the next expected one for the sender
2. The message's timestamp is less than or equal to the local clock.

Conclusion

- (Mattern) Vector Clocks
- Causally Ordered Multicast